

Non-standard neutrinos interactions in a 331 model with minimum Higgs sector

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We present a detailed analysis of a class of extensions to the SM Gauge chiral symmetry $SU(3)_C \times SU(3)_L \times U(1)_X$ (331 model), where the neutrino electroweak interaction with matter via charged and neutral current is modified through new gauge bosons of the model. We found the connections between the non-standard contributions on 331 model with non-standard interactions. Through limits of such interactions in cross section experiments we constrained the parameters of the model, obtaining that the new energy scale of this theory should obey $V > 1.3$ TeV and the new bosons of the model must have masses greater than 610 GeV.

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I. INTRODUCTION

The confirmation of flavor neutrino oscillation [1, 2] by the combination of a variety of data from solar [3–7], atmospheric [8–12] and reactor [13–17] neutrino experiments established the incompleteness of the Standard Model of electroweak interactions, leaving room for other non-standard neutrino properties.

One convenient way to describe neutrino new interactions with matter in the electro-weak (EW) broken phase are the so-called non-standard neutrino interactions (NSI), that is a very widespread and convenient way of parameterizing the effects of new physics in neutrino oscillations [18–23]. NSI with first generation of leptons and quarks for four-fermion operators are contained in the following Lagrangian density [18, 19, 22]:

$$\mathcal{L}_{eff}^{NSI} = -2\sqrt{2}G_F \sum_{f,P} \varepsilon_{\alpha\beta}^{fP} [\bar{f}\gamma^\mu P f] [\bar{\nu}_\alpha \gamma^\mu L \nu_\beta], \quad (1)$$

where G_F is the Fermi constant, $f = u, d, e$ and $P = L, R$ with $2L = (1 - \gamma^5)$, $2R = (1 + \gamma^5)$ and the coefficients $\varepsilon_{\alpha\beta}^{fP}$ encodes the deviation from standard interactions between neutrinos of flavor α with component P -handed of fermions f , resulting in a neutrinos of flavor β . Then, the neutrino oscillations in the presence of non-standard matter effects can be described by an effective Hamiltonian, parameterized as

$$\tilde{H} = \frac{1}{2E} \left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + a \begin{pmatrix} 1 + \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix} \right], \quad (2)$$

where $a = 2\sqrt{2}G_F n_f$, E is the neutrino energy and $\varepsilon_{\alpha\beta} = \sum_{f,P} \varepsilon_{\alpha\beta}^{fP} n_f / n_e$ with n_e and n_f the electrons and fermions f density in the middle, respectively. This parameters $\varepsilon_{\alpha\beta}$ can be found in solar [24], atmospheric [25], accelerator [26] and cross section [27, 28] neutrino data experiment.

We focus on cross section neutrino experiment, where at low energies the standard differential cross section for $\nu_\alpha e \rightarrow \nu_\alpha e$ scattering processes has the well know form:

$$\frac{d\sigma_\alpha}{dT} = \frac{2G_F m_e}{\pi} \left[(g_1^\alpha)^2 + (g_2^\alpha)^2 \left(1 - \frac{T}{E_\nu}\right)^2 - g_1^\alpha g_2^\alpha \frac{m_e T}{E_\nu^2} \right], \quad (3)$$

where m_e is the electron mass, E_ν is the incident neutrino energy, T_e is the electron recoil energy. The quantities g_1^α and g_2^α are related to the SM neutral current couplings of the electron $g_L^e = -1/2 + \sin^2 \theta_W$ and $g_R^e = \sin^2 \theta_W$, with $\sin^2 \theta_W = 0.23119$. For $\nu_{\mu,\tau}$ neutrinos, which take part only in neutral current interactions, we have $g_1^{\mu,\tau} = g_L^e$ and $g_2^{\mu,\tau} = g_R^e$ while for electron neutrinos, which take part in both charge current (CC) and neutral current (NC) interactions, $g_1^e = 1 + g_L^e$, $g_2^e = g_R^e$. In the presence of Non-Universal standard interaction the cross section can be written in the same form of eq. (3) but with $g_{1,2}^\alpha$ replaced by the effective non-standard couplings $\tilde{g}_1^\alpha = g_1^\alpha + \varepsilon_{\alpha\alpha}^{eL}$ and

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$\tilde{g}_2^\alpha = g_2^\alpha + \varepsilon_{\alpha\alpha}^{eR}$, leading to the following differential scattering cross section[27, 28]

$$\frac{d\sigma_\alpha}{dT} = \frac{2G_F m_e}{\pi} \left\{ (g_1^\alpha + \varepsilon_{\alpha\alpha}^{eL})^2 + (g_2^\alpha + \varepsilon_{\alpha\alpha}^{eR})^2 \left(1 - \frac{T_e}{E_\nu}\right)^2 - (g_1^\alpha + \varepsilon_{\alpha\alpha}^{eL}) (g_2^\alpha + \varepsilon_{\alpha\alpha}^{eR}) \frac{m_e T_e}{E_\nu} \right\}. \quad (4)$$

Our goal is to investigate how NSI with matter can be induced by new physics generated by 331 models and based in constraint on this NSI parameters constraint the model some values expected for 331 model. In section II

II. 331 MODEL

The success of the standard model (SM) implies that any new theory should contain the symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ (G_{321}) in a low energy limit. Then, it is natural that one possible modification of SM involves extensions of the representation content in matter and Higgs sector, leading to extension of symmetry group G_{321} to groups $SU(N_C)_C \otimes SU(m)_L \otimes U(1)_X$ with $SU(N_C)_C \otimes SU(m)_L \otimes U(1)_X \supset G_{321}$.

In early 90's, F. Pisano and V. Pleitez [29, 30] and latter P. H. Frampton [31] suggested an extension of the symmetry group $SU(2)_L \otimes U(1)_Y$ of electroweak sector to a group $SU(3)_L \otimes U(1)_X$, *i.e.* with $N_C = m = 3$. The 331 models present some interesting features, as for instance, they associate the number of families to internal consistence of the theory, preserving asymptotic freedom.

In these models, the SM doublets are part of triplets. In quark sector three new quarks are included to build the triplets, while in lepton sector we can use the right-handed neutrino to such role[29, 31]. Another option is to invoke three new heavy leptons, charged or not, depending on the choice of charge operator[32, 33]. In SM the electric charge operator is constructed as a combination of diagonal generators of $SU(2) \otimes U(1)_Y$. Then, it is natural to assume that this operator in $SU(3)_L \otimes U(1)_X$ is defined in the same way. The most general charge operator in $SU(3)_L \otimes U(1)_X$ is a linear superposition of diagonal generators of symmetry groups, given by:

$$\mathcal{Q} \equiv aT_{3L} + \frac{2}{\sqrt{3}}bT_{8L} + XI_3, \quad (5)$$

where the group generator are defined as $T_{iL} \equiv \lambda_{iL}/2$ with λ_{iL} , $i = 1, \dots, 8$ are Gell-Mann matrices for $SU(3)_L$, where the normalization chosen is $Tr(\lambda_{iL}\lambda_{jL}) = 2\delta_{ij}$ and $I_3 = \text{diag}(1, 1, 1)$ is the identity matrix, a and b are two parameters to be determined. Then, eq. (5) in representation 3 have the form:

$$\mathcal{Q}[3] = \begin{pmatrix} \frac{a}{2} + \frac{b}{3} + X & 0 & 0 \\ 0 & -\frac{a}{2} + \frac{b}{3} + X & 0 \\ 0 & 0 & -\frac{2b}{3} + X \end{pmatrix}, \quad (6)$$

where we have two free parameters to obtain the charge of fermions, a and b (X can be determined by anomalies cancellation). However, $a = 1$ is necessary to obtain doublets of isospins $SU(2) \otimes U(1)_Y$ correctly incorporated in the model $SU(3)_L \otimes U(1)_X$ [32–34]. Then we can vary b to create different models in 331 context, being a signature which differentiate such models. For $b = -3/2$, we have the original 331 model[29, 30].

To have local gauge invariance we have the following covariant derivative: $D_\mu = \partial_\mu - i\frac{g}{2}\lambda_\alpha W_\mu^\alpha - ig_x X B_\mu$ and a total of 17 mediator bosons: one field B_μ associated with $U(1)_X$, eight fields associated with $SU(3)_C$, and another eight fields associated with $SU(3)_L$, written in the form:

$$\mathbf{W}_\mu \equiv W_\mu^\alpha \lambda_\alpha = \begin{pmatrix} W_\mu^3 + \frac{1}{\sqrt{3}}W_\mu^8 & \sqrt{2}W_\mu^+ & \sqrt{2}K_\mu^{\mathcal{Q}_1} \\ \sqrt{2}W_\mu^- & -W_\mu^3 + \frac{1}{\sqrt{3}}W_\mu^8 & \sqrt{2}K_\mu^{\mathcal{Q}_2} \\ \sqrt{2}K_\mu^{-\mathcal{Q}_1} & \sqrt{2}K_\mu^{-\mathcal{Q}_2} & -\frac{2}{\sqrt{3}}W_\mu^8 \end{pmatrix}, \quad (7)$$

where

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_{1\mu} \mp iW_{2\mu}), \quad (8)$$

$$K_\mu^{\pm\mathcal{Q}_1} = \frac{1}{\sqrt{2}}(W_{4\mu} \mp iW_{5\mu}), \quad (9)$$

$$K_\mu^{\pm\mathcal{Q}_2} = \frac{1}{\sqrt{2}}(W_{6\mu} \mp iW_{7\mu}), \quad (10)$$

Therefore, charge operator in eq. (6) applied over eq. (7) leads to:

$$\mathcal{Q}_W \rightarrow \begin{pmatrix} 0 & +1 & \frac{1}{2} + b \\ -1 & 0 & -\frac{1}{2} + b \\ \frac{1}{2} + b & -\frac{1}{2} + b & 0 \end{pmatrix}. \quad (11)$$

The mediator bosons will have integer electric charge only if $b = \pm 1/2, \pm 3/2, \pm 5/2, \dots, \pm(2n+1)/2, n = 0, 1, 2, 3, \dots$. A detailed analysis shows that if b is associated with the fundamental representation 3 then $-b$ will be associated with antisymmetric representation 3^* .

A. The representation content

There are many representations for the matter content[35], for instance $b = 3/2$ [29]. But we note that if we accommodate the doublets of $SU(2)_L$ in the superior components of triplets and anti-triplets of $SU(3)_L$, and if we forbid exotic charges for the new fermions, we obtain from eq. (6) the constrain $b = \pm 1/2$ (assuming $a = 1$). Since a negative value of b can be associated to the anti-triplet, we obtain that $b = 1/2$ is a necessary and sufficient condition to exclude exotic electric charges in fermion and boson sector[32].

The fields left and right-handed components transform under $SU(3)_L$ as triplets and singlets, respectively. Therefore the theory is quiral and can present anomalies of Alder-Bell-Jackiw[36–38]. In a non-abelian theory, in the fermionic representation \mathcal{R} the divergent anomaly is given by:

$$\mathcal{A}^{abc} \propto \sum_{\mathcal{R}} Tr [\{T_L^a(\mathcal{R}), T_L^b(\mathcal{R})\} T_L^c(\mathcal{R}) - \{T_R^a(\mathcal{R}), T_R^b(\mathcal{R})\} T_R^c(\mathcal{R})], \quad (12)$$

where $T^a(\mathcal{R})$ are symmetry group matrixial representation. The indexes R and L relate to the quiral property of the fields. Therefore, to eliminate the pure anomaly $[SU(3)_L]^3$ we should have that $\mathcal{A}^{abc} \propto \sum_{\mathcal{R}'} Tr [\{T_L^a(\mathcal{R}'), T_L^b(\mathcal{R}')\} T_L^c(\mathcal{R}')] = 0$. We use the fact that $SU(3)_L$ has two fundamental representations, 3 and 3^* , where $T^{a*} = -T^a$, which is equivalent to say that $T_L^{a*}(\mathcal{R}^*) = -T_L^a(\mathcal{R})$ [39]. Then:

$$\begin{aligned} & \sum_{\mathcal{R}'} Tr [\{T_L^a(\mathcal{R}'), T_L^b(\mathcal{R}')\} T_L^c(\mathcal{R}')] \\ &= \sum_{\mathcal{R}} Tr [\{T_L^a(\mathcal{R}), T_L^b(\mathcal{R})\} T_L^c(\mathcal{R})] + \sum_{\mathcal{R}^*} Tr [\{T_L^{a*}(\mathcal{R}^*), T_L^{b*}(\mathcal{R}^*)\} T_L^{c*}(\mathcal{R}^*)] \end{aligned} \quad (13)$$

$$= \sum_{\mathcal{R}} Tr [\{T_L^a(\mathcal{R}), T_L^b(\mathcal{R})\} T_L^c(\mathcal{R})] - \sum_{\mathcal{R}} Tr [\{T_L^a(\mathcal{R}), T_L^b(\mathcal{R})\} T_L^c(\mathcal{R})]. \quad (14)$$

So, we can see that for the anomalies to be canceled, the number of fields that transform as triplets (first term in eq. (14)) and anti-triplets under $SU(3)_L$ has to be the same. This implies that two families of quarks should transform different then the third family, as will be discussed in next section.

Usually the third quark family is chosed to transform in a different way that the first two families. But we will assume that the first family transform differently, to address the fact that $m_u < m_d, m_{\nu_e} < m_\ell$ while $m_c \gg m_s$ and $m_t \gg m_b$. To state in a more clear way, we remember that in SM the $SU(2)_L$ doublets are: $(\nu_\ell, \ell)^T, (u, d)^T, (c, s)^T, (t, b)^T$, with $\ell = e, \mu, \tau$. We can see that the first component of leptons doublets and first quark family is lighter than the second component. But for the second and third quark families is the opposite. Then we use this idea to justify that first quark family transform as leptons.

B. Minimal 331 model on scalar sector

Among the different possibilities of 331 models, we will present a detailed study on a Minimal Model on scalar sector without exotic electric charges for quarks and with three news leptons without charged [32] ($b = 1/2$), where the fermions present the following transformation structure under $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$:

$$\begin{aligned} \psi_{\ell L} &= (\ell^-, \nu_\ell, N_\ell^0)_L^T \sim (1, 3^*, -1/3), \\ \nu_{\ell R} &\sim (1, 1, 0), \\ \ell_R^- &\sim (1, 1, -1), \\ N_{\ell R}^0 &\sim (1, 1, 0), \end{aligned}$$

$$\begin{aligned}
Q_{1L} &= (d, u, U_1)_L^T \sim (3, 3^*, 1/3), \\
u_{iR} &\sim (3, 1, 2/3), \\
d_{iR} &\sim (3, 1, -1/3), \\
U_{1R} &\sim (3, 1, 2/3), \\
Q_{aL} &= (u_a, d_a, D_a)_L^T \sim (3, 3, 0), \\
D_{aR} &\sim (3, 1, -1/3),
\end{aligned} \tag{15}$$

where $i = 1, 2, 3$, $\ell = e, \mu, \tau$, $a = 2, 3$. We note that the leptons multiplets $\psi_{\ell L}$ consist of three fields $\ell = \{e, \mu, \tau\}$, the corresponding neutrinos $\nu_\ell = \{\nu_e, \nu_\mu, \nu_\tau\}$ and new neutral leptons $N_\ell^0 = \{N_e^0, N_\mu^0, N_\tau^0\}$. We can also see that the multiplet associated with the first quark family Q_{1L} consists of quarks down, up and a new quark with the same electric charge of quark up (named U_1), while the multiplet associated with second (third) family Q_{aL} consist of SM quarks of second (third) family and a new quark with the same electric charge of quark down (named D_2 (D_3)). The numbers on parenthesis refer to the transformation properties under $SU(3)_C$, $SU(3)_L$ and $U(1)_X$ respectively. With this choice the anomalies cancel in a non-trivial way[40], and asymptotic freedom is guaranteed[41, 42].

1. Scalar sector and Yukawa couplings

The scalar fields have to be coupled to fermions by Yukawa terms, invariants under $SU(3)_L \otimes U(1)_X$. In lepton sector, these couplings can be written as:

$$\begin{aligned}
\bar{\psi}_{\ell L} \ell_R &\sim (1, 3, 1/3) \otimes (1, 1, -1) = \underbrace{(1, 3, -2/3)}_{\rho^*}, \\
\bar{\psi}_{\ell L} \nu_{\ell R} &\sim (1, 3, 1/3) \otimes (1, 1, 0) = \underbrace{(1, 3, 1/3)}_{\eta}, \\
\bar{\psi}_{\ell L} N_{\ell R}^0 &\sim (1, 3, 1/3) \otimes (1, 1, 0) = \underbrace{(1, 3, 1/3)}_{\chi},
\end{aligned} \tag{16}$$

and in quarks sector:

$$\begin{aligned}
\bar{Q}_{1L} u_{iR} &= (3^*, 3, -1/3) \otimes (3, 1, 2/3) = \underbrace{(1, 3, 1/3)}_{\chi} \oplus \underbrace{(8, 3, 1/3)}_{\text{Colour Higgs}}, \\
\bar{Q}_{1L} d_{iR} &= (3^*, 3, -1/3) \otimes (3, 1, -1/3) = \underbrace{(1, 3, -2/3)}_{\rho^*} \oplus \dots, \\
\bar{Q}_{1L} U_{1R} &= (3^*, 3, -1/3) \otimes (3, 1, 2/3) = \underbrace{(1, 3, 1/3)}_{\chi} \oplus \dots, \\
\bar{Q}_{aL} u_{iR} &= (3^*, 3^*, 0) \otimes (3, 1, 2/3) = \underbrace{(1, 3^*, 2/3)}_{\rho} \oplus \dots, \\
\bar{Q}_{aL} d_{iR} &= (3^*, 3^*, 0) \otimes (3, 1, -1/3) = \underbrace{(1, 3^*, -1/3)}_{\eta^*} \oplus \dots, \\
\bar{Q}_{aL} D_{aR} &= (3^*, 3^*, 0) \otimes (3, 1, -1/3) = \underbrace{(1, 3^*, -1/3)}_{\chi^*} \oplus \dots,
\end{aligned} \tag{17}$$

As usual in these class of models, we impose non-colored Higgs, selecting only the multiplets that transform as singlets under $SU(3)_C$. We note that we need only three Higgs multiplets ρ , χ and η , to couple the different fermionic fields and generate mass through spontaneous symmetry breaking. In eqs. (16) and (17) we note that quantum numbers of triplets χ and η are the same, which leads us to consider models with two or three Higgs triplets. We will adopt the first option, two Higgs triplets, due to the simpler scalar sector in comparison with the scenario with three triplets[32, 33].

C. Model with two Higgs triplets

For the models with two Higgs triplets, we obtain¹

$$\begin{aligned}\Phi_1 &= (\phi_1^-, \phi_1'^0, \phi_1^0)^T \sim (1, 3^*, -1/3), \\ \Phi_2 &= (\phi_2^0, \phi_2^+, \phi_2'^+)^T \sim (1, 3^*, 2/3).\end{aligned}\quad (18)$$

Assuming the following choice to the Higgs triplets vacuum expectation value (VEV) [32] $\langle \Phi_1 \rangle_0 = (0, \vartheta_1, V)^T$ and $\langle \Phi_2 \rangle_0 = (\vartheta_2, 0, 0)^T$ we associate V with the mass of the new fermions, which lead us to assume $V \gg \vartheta_1, \vartheta_2$. We expand the scalar VEV's in the following way:

$$\phi_1^0 = V + \frac{H_{\phi_1}^0 + iA_{\phi_1}^0}{\sqrt{2}}, \quad \phi_1'^0 = \vartheta_1 + \frac{H_{\phi_1}'^0 + iA_{\phi_1}'^0}{\sqrt{2}}, \quad \phi_2^0 = \vartheta_2 + \frac{H_{\phi_2}^0 + iA_{\phi_2}^0}{\sqrt{2}}. \quad (19)$$

The real (imaginary) part H_{ϕ_i} (A_{ϕ_i}) is usually called CP-even (CP-odd) scalar field. The most general potential can be written as:

$$\begin{aligned}V(\Phi_1, \Phi_2) &= \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ &\quad + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1).\end{aligned}\quad (20)$$

Demanding that in the displaced potential $V(\Phi_1, \Phi_2)$ the linear terms on the field should be absent, we have, in tree level approximation, the following constraints:

$$\begin{aligned}\mu_1^2 + 2\lambda_1 (\vartheta_1^2 + V^2) + \lambda_3 \vartheta_2^2 &= 0, \\ \mu_2^2 + \lambda_3 (\vartheta_1^2 + V^2) + 2\lambda_2 \vartheta_2^2 &= 0.\end{aligned}\quad (21)$$

The analysis of such equations shows that they are related to a minimum in scalar potential with the value:

$$V_{min} = -\vartheta_2^4 \lambda_2 - (\vartheta_1^2 + V^2) [(\vartheta_1^2 + V^2) \lambda_1 + \vartheta_2^2 \lambda_3] = V(\vartheta_1, \vartheta_2, V). \quad (22)$$

Replacing eq. (19) and (21) in eq. (20) we can calculate the mass matrix in $(H_{\phi_1}^0, H_{\phi_2}^0, H_{\phi_1}'^0)$ basis through the relation $M_{ij}^2 = 2 \frac{\partial^2 V(\Phi_1, \Phi_2)}{\partial H_{\phi_i}^0 \partial H_{\phi_j}^0}$, obtaining:

$$M_H^2 = 2 \begin{pmatrix} 2\lambda_1 V^2 & \lambda_3 \vartheta_2 V & 2\lambda_1 \vartheta_1 V \\ \lambda_3 \vartheta_2 V & 2\lambda_2 \vartheta_2^2 & \lambda_3 \vartheta_1 \vartheta_2 \\ 2\lambda_1 \vartheta_1 V & \lambda_3 \vartheta_1 \vartheta_2 & 2\lambda_1 \vartheta_1^2 \end{pmatrix}. \quad (23)$$

Since eq. (23) has vanishing determinant, we have one Goldstone boson G_1 and two massive neutral scalar fields H_1 and H_2 with masses ²

$$\begin{aligned}M_{H_1, H_2}^2 &= 2\lambda_1 (\vartheta_1^2 + V^2) + 2\lambda_2 \vartheta_2^2 \\ &\quad \pm 2\sqrt{[\lambda_1 (\vartheta_1^2 + V^2) + \lambda_2 \vartheta_2^2]^2 + \vartheta_2^2 (\vartheta_1^2 + V^2) (\lambda_3^2 - 4\lambda_1 \lambda_2)},\end{aligned}\quad (24)$$

where real values for λ 's produce positive mass to neutral scalar fields only if $\lambda_1 > 0$ and $4\lambda_1 \lambda_2 > \lambda_3^2$, which implies that $\lambda_2 > 0$. A detailed analysis shows that when $V(\Phi_1, \Phi_2)$ in eq. (20) is expanded around the most general vacuum, given by eq. (19) and using constrains in eq. (21), we don't obtain pseudo-scalar fields $A_{\phi_i}^0$. This allows us to identify

¹ Note that in this model we assumed $\Phi_1 = \chi, \eta$ e $\Phi_2 = \rho$

² Note that if $\lambda_3^2 = 4\lambda_1 \lambda_2$ we obtain two Goldstone bosons, G_1 and H_2 , and a massive scalar field H_1 with mass $M_{H_1}^2 = 4[\lambda_1 (\vartheta_1^2 + V^2) + \lambda_2 \vartheta_2^2]$ where $\lambda_1 \lambda_2 > 0$, then imposing $M_{H_1}^2 > 0$ leads to $\lambda_1 > 0$ and $\lambda_2 > 0$.

three more Goldstone bosons $G_2 = A_{\Phi_1}^0$, $G_3 = A_{\Phi_2}^0$ and $G_4 = A_{\Phi_1}^{\prime 0}$. For the mass spectrum in charged scalar sector on $(\phi_1^-, \phi_2^+, \phi_2^{\prime +})$ basis the mass matrix will be given by:

$$M_+^2 = 2\lambda_4 \begin{pmatrix} \vartheta_2^2 & \vartheta_1\vartheta_2 & \vartheta_2 V \\ \vartheta_1\vartheta_2 & \vartheta_1^2 & \vartheta_1 V \\ \vartheta_2 V & \vartheta_1 V & V^2 \end{pmatrix}, \quad (25)$$

with two eigenvalues equal to zero, equivalent to four Goldstone bosons G_5^\pm , G_6^\pm and two physical charged scalar fields with large masses given by $\lambda_4 (\vartheta_1^2 + \vartheta_2^2 + V^2)$, which leads to the constrain $\lambda_4 > 0$.

This analysis shows that, after symmetry breaking, the original twelve degrees of freedom in scalar sector leads to eight Goldstone bosons (four electrically neutral and four electrically charged), four physical scalar fields, two neutral (one of which being the SM Higgs scalar) and two charged. Eight Goldstone bosons should be absorbed by eight gauge fields as we will see in next section.

1. Gauge Sector with two Higgs triplets

The gauge bosons interaction with matter in electroweak sector appears with the covariant derivative for a matter field φ as:

$$D_\mu^\varphi = \partial_\mu - \frac{i}{2}gW_\mu^a \lambda_{aL} - ig_X X_\varphi B_\mu = \partial_\mu - \frac{i}{2}g\mathcal{M}_\mu^\varphi, \quad (26)$$

where λ_{aL} , $a = 1, \dots, 8$ are Gell-Mann matrices of $SU(3)_L$ algebra, and X_φ is the charge of abelian factor $U(1)_X$ of the multiplet φ in which D_μ acts. The matrix \mathcal{M}_μ^φ contain the gauge bosons with electric charges q , defined by the generic charge operator in eq. (5) and eq. (11). For $b = 1/2$ the matrix \mathcal{M}_μ^φ will have the form:

$$\mathcal{M}_\mu^\varphi = \begin{pmatrix} W_{3\mu} + \frac{W_{8\mu}}{\sqrt{3}} + 2tX_\varphi B_\mu & \sqrt{2}W_\mu^+ & \sqrt{2}K_\mu^+ \\ \sqrt{2}W_\mu^- & -W_{3\mu} + \frac{W_{8\mu}}{\sqrt{3}} + 2tX_\varphi B_\mu & \sqrt{2}K_\mu^0 \\ \sqrt{2}K_\mu^- & \sqrt{2}\bar{K}_\mu^0 & \frac{-2W_{8\mu}}{\sqrt{3}} + 2tX_\varphi B_\mu \end{pmatrix}, \quad (27)$$

where $t = g_x/g$ and non physical gauge bosons on non-diagonal entries, W_μ^\pm and K_μ^\pm , are defined in eq. (8) and (9) with $Q_1 = 1$ respectively, and:

$$K_\mu^0 = \frac{1}{\sqrt{2}}(A_{6\mu} - iA_{7\mu}), \quad (28)$$

$$\bar{K}_\mu^0 = \frac{1}{\sqrt{2}}(A_{6\mu} + iA_{7\mu}). \quad (29)$$

Then for the 331 model we are considering ($b = 1/2$) we have two neutral gauge bosons, K_μ^0 and \bar{K}_μ^0 , and four charged gauge bosons, W_μ^\pm and K_μ^\pm . The three physical neutral eigenstates will be a linear combination of $W_{3\mu}$, $W_{8\mu}$ and B_μ . After breaking the symmetry with $\langle \Phi_i \rangle$, $i = 1, 2$, and using covariant derivative $D_\mu = \partial_\mu - \frac{i}{2}g\mathcal{M}_\mu^\varphi$ for the triplets Φ_i we obtain the following masses for the charged physical fields:

$$M_{W'}^2 = \frac{1}{2}g^2\vartheta_2^2, \quad M_{K'}^2 = \frac{1}{2}g^2(\vartheta_1^2 + \vartheta_2^2 + V^2), \quad (30)$$

and the following physical eigenstates:

$$W_\mu^{\prime \pm} = \frac{1}{\sqrt{\vartheta_1^2 + V^2}}(-\vartheta_1 K_\mu^\pm + V W_\mu^\pm), \quad K_\mu^{\prime \pm} = \frac{1}{\sqrt{\vartheta_1^2 + V^2}}(V K_\mu^\pm + \vartheta_1 W_\mu^\pm). \quad (31)$$

The neutral sector in approximation $(\frac{\vartheta_i}{V})^n \approx 0$ for $n > 2$ leads to the following masses for the neutral physical fields:

$$M_{foton}^2 = 0, \\ M_{Z'}^2 = \frac{1}{2}g^2(V^2 + \vartheta_1^2),$$

$$\begin{aligned}
M_Z^2 &\approx \frac{1}{2}g^2\vartheta_2^2 \left(\frac{3g^2 + 4g_x^2}{3g^2 + g_x^2} \right), \\
M_{K_R^0}^2 &\approx \frac{2}{9}(V^2 + \vartheta_1^2)(3g^2 + g_x^2) + \frac{\vartheta_2^2(3g^2 + 4g_x^2)^2}{18(3g^2 + g_x^2)}, \\
M_{K_I^0}^2 &= \frac{1}{2}g^2(V^2 + \vartheta_1^2).
\end{aligned} \tag{32}$$

We can see from eqs. (30) and (32) that we have one non-massive boson, which we associate with the photon, and four massive neutral fields, where the mass of one of them is proportional to ϑ_2 while the other three have masses proportional to V (new energy scale). Therefore we can associate the field Z with SM Z_μ , and the fields Z' , K_I^0 and K_R^0 , with three new neutral bosons. We also have four massive charged fields, where two of them have masses proportional to ϑ_2 . Therefore we can associate the fields $W_\mu'^\pm$ to the SM fields W_μ^\pm , while the fields $K_\mu'^\pm$ are new bosons. The eigenstates B_μ , $W_{3\mu}$, $W_{8\mu}$ and $K_{R\mu}^o$ can be related to the physical eigenstates A_μ , $Z_\mu'^0$, Z_μ^0 and $K_{R\mu}^{'0}$ by:

$$\begin{pmatrix} B_\mu \\ W_{3\mu} \\ W_{8\mu} \\ K_{R\mu}^o \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} A_\mu \\ Z_\mu'^0 \\ Z_\mu^0 \\ K_{R\mu}^{'0} \end{pmatrix}. \tag{33}$$

Assuming $(\frac{\vartheta_i}{V})^n \sim 0$ for $n > 2$, we obtain

$$\mathbf{M}^{-1} = \begin{pmatrix} -\frac{1}{t}S_W & 0 & \frac{1}{t}T_W^2C_W + \beta_1 & -\frac{1}{\sqrt{3}}T_W + \beta_2 \\ S_W & \frac{-\vartheta_1}{V} & C_W + \beta_3 & \beta_4 \\ \frac{1}{\sqrt{3}}S_W & \frac{\sqrt{3}\vartheta_1}{V} & -\frac{1}{\sqrt{3}}T_WS_W + \beta_5 & -\frac{1}{t}T_W + \beta_6 \\ 0 & 1 - \beta_7 & \frac{\vartheta_1}{V}C_W^{-1} & \frac{\sqrt{3}\vartheta_1}{tV}T_W \end{pmatrix}, \tag{34}$$

where, again, $t = g_x/g$ and:

$$\begin{aligned}
S_W &= \frac{\sqrt{3}g_x}{\sqrt{3g^2 + 4g_x^2}}, \quad C_W = \sqrt{1 - S_W^2}, \quad T_W = \frac{S_W}{C_W}, \\
\beta_1 &= -\frac{\vartheta_2^2}{4tV^2}T_W^2C_W^{-3}, \quad \beta_2 = -\frac{\sqrt{3}\vartheta_2^2}{4t^2V^2}T_W^3C_W^{-2}, \\
\beta_3 &= -\frac{\vartheta_1^2}{2V^2}C_W^{-1}, \quad \beta_4 = -\frac{\sqrt{3}(2C_W^2\vartheta_1^2 + \vartheta_2^2)}{4tV^2}T_WC_W^{-2}, \\
\beta_5 &= \frac{6C_W^4\vartheta_1^2 - (3 - 4S_W^2)\vartheta_2^2}{4\sqrt{3}V^2C_W^5}, \quad \beta_6 = \frac{(6C_W^4\vartheta_1^2 + S_W^2\vartheta_2^2)}{4tV^2C_W^4}T_W, \\
\beta_7 &= -\frac{2\vartheta_2^2}{V^2}.
\end{aligned} \tag{35}$$

We note that all β_i are of order $\mathcal{O}\left(\left(\frac{\vartheta_i}{V}\right)^2\right)$. So, assuming $\vartheta_i \sim \mathcal{O}(10^{-1})$ TeV, for a new energy scale of order $V \sim 10$ TeV all the β_i 's are negligible.

2. Charged and Neutral Currents

The interaction between gauge bosons and fermions in flavor basis is given by the following Lagrangian density:

$$\mathcal{L}_f = \bar{R}i\gamma^\mu(\partial_\mu + ig_x B_\mu X_R)R + \bar{L}i\gamma^\mu(\partial_\mu + ig_x B_\mu X_L + \frac{ig}{2}\lambda_a W_\mu^a)L, \tag{36}$$

where R represents any right-handed singlet and L any left-handed triplet. We can write $\mathcal{L}_f = \mathcal{L}_{lep} + \mathcal{L}_{Q_1} + \mathcal{L}_{Q_a}$ and in lepton sector we obtain:

$$\mathcal{L}_{lep} = \mathcal{L}_{lep}^{kin} + \mathcal{L}_{lep}^{CC} + \mathcal{L}_{lep}^{NC}, \tag{37}$$

where

$$\mathcal{L}_{lep}^{kin} = \bar{R}i\gamma^\mu\partial_\mu R + \bar{L}i\gamma^\mu\partial_\mu L, \quad (38)$$

$$\mathcal{L}_{lep}^{CC} = -\frac{g}{\sqrt{2}}\bar{\ell}_L\gamma^\mu\nu_{\ell L}W_\mu^+ - \frac{g}{\sqrt{2}}\bar{\ell}_L\gamma^\mu N_{\ell L}^0 K_\mu^+ + h.c., \quad (39)$$

$$\begin{aligned} \mathcal{L}_{lep}^{NC} = & \frac{g_x}{3} \left[\bar{\ell}_L\gamma^\mu\ell + \overline{\nu_{\ell L}}\gamma^\mu\nu_{\ell L} + \overline{N_{\ell L}^0}\gamma^\mu N_{\ell L}^0 \right] B_\mu + g_x\bar{\ell}_R\gamma^\mu\ell_R B_\mu \\ & - \frac{g}{2\sqrt{3}} \left[\bar{\ell}_L\gamma^\mu\ell_L + \overline{\nu_{\ell L}}\gamma^\mu\nu_{\ell L} - 2t\overline{N_{\ell L}^0}\gamma^\mu N_{\ell L}^0 \right] W_{8\mu} - \frac{g}{\sqrt{2}}\overline{\nu_{\ell L}}\gamma^\mu N_{\ell L}^0 K^{0\mu} \\ & - \frac{g}{2} \left[\bar{\ell}_L\gamma^\mu\ell_L - \overline{\nu_{\ell L}}\gamma^\mu\nu_{\ell L} \right] W_{3\mu} - \frac{g}{\sqrt{2}}\overline{N_{\ell L}^0}\gamma^\mu\nu_{\ell L}\bar{K}_\mu^0. \end{aligned} \quad (40)$$

In quark sector we have that for the first family triplet $X = 1/3$, and for the singlets d , u , and U_1 we have $X = -1/3$, $2/3$ and $2/3$, respectively. Then we have

$$\mathcal{L}_{Q_1}^{kin} = \bar{Q}_{1R}i\gamma^\mu\partial_\mu Q_{1R} + \bar{Q}_{1L}i\gamma^\mu\partial_\mu Q_{1L}, \quad (41)$$

$$\mathcal{L}_{Q_1}^{CC} = -\frac{g}{\sqrt{2}}\bar{d}_L\gamma^\mu u_L W_\mu^+ - \frac{g}{\sqrt{2}}\bar{d}_L\gamma^\mu U_{1L} K_\mu^+ + h.c., \quad (42)$$

$$\begin{aligned} \mathcal{L}_{Q_1}^{NC} = & \frac{g_x}{3} (\bar{d}_R\gamma^\mu d_R - 2\bar{u}_R\gamma^\mu u_R - 2\bar{U}_{1R}\gamma^\mu U_{1R}) B_\mu + \frac{g}{2}\bar{u}_L\gamma^\mu u_L W_{3\mu} \\ & - \frac{g_x}{3} (\bar{d}_L\gamma^\mu d_L + \bar{u}_L\gamma^\mu u_L + \bar{U}_{1L}\gamma^\mu U_{1L}) B_\mu - \frac{g}{2}\bar{d}_L\gamma^\mu d_L W_{3\mu} - \frac{g}{\sqrt{2}}\bar{U}_{1L}\gamma^\mu u_L \bar{K}_\mu^0 \\ & - \frac{g}{2\sqrt{3}} (\bar{d}_L\gamma^\mu d_L + \bar{u}_L\gamma^\mu u_L - 2\bar{U}_{1L}\gamma^\mu U_{1L}) W_{8\mu} - \frac{g}{\sqrt{2}}\bar{u}_L\gamma^\mu U_{1L} K_\mu^0. \end{aligned} \quad (43)$$

For second and third families we know that $X = 0$ for the triplets and $X = 2/3$, $-1/3$ and $-1/3$ for the singlets $u_{2,3}$, $d_{2,3}$, $D_{2,3}$, respectively, where $u_2 = c$, $u_3 = t$, $d_2 = s$, $d_3 = b$. Then we obtain for $a = 2, 3$:

$$\mathcal{L}_{Q_a}^{kin} = \bar{Q}_{aR}i\gamma^\mu\partial_\mu Q_{aR} + \bar{Q}_{aL}i\gamma^\mu\partial_\mu Q_{aL}, \quad (44)$$

$$\mathcal{L}_{Q_a}^{CC} = -\frac{g}{\sqrt{2}}\bar{u}_{aL}\gamma^\mu d_{aL} W_\mu^+ - \frac{g}{\sqrt{2}}\bar{u}_{aL}\gamma^\mu D_{aL} K_\mu^+ + h.c., \quad (45)$$

$$\begin{aligned} \mathcal{L}_{Q_a}^{NC} = & \frac{g_x}{3} [-2\bar{u}_{aR}\gamma^\mu u_{aR} + \bar{d}_{aR}\gamma^\mu d_{aR} + \bar{D}_{aR}\gamma^\mu D_{aR}] B_\mu \\ & - \frac{g}{2\sqrt{3}} [\bar{u}_{aL}\gamma^\mu u_{aL} + \bar{d}_{aL}\gamma^\mu d_{aL} - 4\bar{D}_{aL}\gamma^\mu D_{aL}] W_{8\mu} - \frac{g}{\sqrt{2}}\bar{d}_{aL}\gamma^\mu D_{aL} K_\mu^0 \\ & - \frac{g}{2} [\bar{u}_{aL}\gamma^\mu u_{aL} - \bar{d}_{aL}\gamma^\mu d_{aL}] W_{3\mu} - \frac{g}{\sqrt{2}}\bar{D}_{aL}\gamma^\mu d_{aL} \bar{K}_\mu^0. \end{aligned} \quad (46)$$

III. NEUTRINOS INTERACTIONS WITH MATTER IN 331 MODEL

It is well known that neutrino oscillation phenomenon in a material medium, as the sun, earth or in a supernova, can be quite different from the oscillation that occurs in vacuum, since the interactions in the medium modify the dispersion relations of the particles traveling through it [43]. From the macroscopic point of view, the modifications of neutrino dispersion relations can be represented in terms of a refractive index or an effective potential. And according to [43, 44], the effective potential can be calculated from the amplitudes of coherent elastic scattering in relativistic limit.

In the present 331 model, the coherent scattering will be induced by neutral currents, NC, mediated by bosons Z_μ^0 , Z_μ^0 , and $K_{R\mu}^0$ and by charged currents, CC, mediated by bosons W_μ^\pm and K_μ^\pm . Following [44], we calculate in next sections the neutrino effective potentials in coherent scattering.

A. Charged Currents

The first term of eq. (39) shows that the interaction of charged leptons with neutrinos occurs only through the gauge bosons W_μ^\pm , then by eq. (31) we obtain that the interaction through charged bosons is given by:

$$-\frac{g}{\sqrt{2}}\bar{\ell}_L\gamma^\mu\nu_{\ell L}W_\mu^\pm = -\frac{Vg}{\sqrt{2}\sqrt{\vartheta_1^2 + V^2}}\bar{\ell}_L\gamma^\mu\nu_{\ell L}W_\mu^{\prime\pm} - \frac{g\vartheta_1}{\sqrt{2}\sqrt{\vartheta_1^2 + V^2}}\bar{\ell}_L\gamma^\mu\nu_{\ell L}K_\mu^{\prime\pm}. \quad (47)$$

The amplitude for the neutrino elastic scattering with charged leptons in tree level through CC is given by ³

$$\begin{aligned}\mathcal{L}_{int}^{cc} = & - \left(-\frac{Vg}{\sqrt{2}\sqrt{\vartheta_1^2 + V^2}} \right)^2 \bar{\ell}_L(p_1)\gamma^\mu\nu_{\ell L}(p_2) \frac{-ig_{\mu\lambda}}{(p_2 - p_1)^2 - M_{W'}^2} \bar{\nu}_{\ell L}(p_3)\gamma^\lambda\ell_L(p_4) \\ & - \left(-\frac{g\vartheta_1}{\sqrt{2}\sqrt{\vartheta_1^2 + V^2}} \right)^2 \bar{\ell}_L(p_1)\gamma^\mu\nu_{\ell L}(p_2) \frac{-ig_{\mu\lambda}}{(p_2 - p_1)^2 - M_{K'}^2} \bar{\nu}_{\ell L}(p_3)\gamma^\lambda\ell_L(p_4).\end{aligned}\quad (48)$$

For low energies $M_{W'}^2, M_{K'}^2 \gg (p_2 - p_1)^2$, the effective Lagrangian is given by:

$$\mathcal{L}_{eff}^{cc} \approx - \frac{g^2}{2(\vartheta_1^2 + V^2)} \left(\frac{V^2}{M_{W'}^2} + \frac{\vartheta_1^2}{M_{K'}^2} \right) [\bar{\ell}_L(p_1)\gamma^\mu\ell_L(p_4)] [\bar{\nu}_{\ell L}(p_3)\gamma_\mu\nu_{\ell L}(p_2)], \quad (49)$$

where we used Fierz transformation[?] to go from eq. (48) to eq. (49). Replacing eq. (30) in eq. (49) we obtain:

$$-\mathcal{L}_{eff}^{cc} \approx \left[\frac{1}{\vartheta_2^2} - \frac{\vartheta_1^2}{V^2\vartheta_2^2} + \left(\frac{\vartheta_1^2}{V^4} \right)_{K'} + \mathcal{O}\left(\frac{1}{V^4}\right) \right] \left\langle \bar{\ell}\gamma^\mu \frac{(1 - \gamma_5)}{2} \ell \right\rangle \{ \bar{\nu}_{\ell L}(p)\gamma^\mu\nu_{\ell L}(p) \}, \quad (50)$$

where we used $()_{K'}$ to denote the term that appears from the new charged boson. We can see that for a new energy scale $V \gg \vartheta_1$ the term that comes from the new boson does not contribute to the process, as expected, since the new charged boson K_{μ}^{\pm} has a mass of the order of the new energy scale of the theory (see eq. (30)).

Now, since usual matter has only leptons from first family, we will restrain our calculations to the neutrino interactions with first family standard model particles. The term $\langle \rangle$ in eq. (50) can be calculated following [44], where we have the correspondence $\langle \bar{e}\gamma^\mu\gamma_5 e \rangle \sim \text{spin}$, $\langle \bar{e}\gamma_i e \rangle \sim \text{velocity}$ and $\langle \bar{e}\gamma_0 e \rangle \sim n_e$, where n_e is the electronic density. Assuming non-polarized medium and vanishing average velocity, we obtain that eq. (50) can be written as:

$$\mathcal{L}_{eff}^{cc} \approx - \left[\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} + \left(\frac{\vartheta_1^2}{2V^4} \right)_{K'} + \mathcal{O}(V^{-4}) \right] n_e \bar{\nu}_{eL}\gamma^\mu\nu_{eL}. \quad (51)$$

The modifications on electronic neutrino dispersion relations can be represented by the following effective potential:

$$V_{CC}^e \approx \frac{1}{2\vartheta_2^2} n_e - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} n_e + \left(\frac{\vartheta_1^2}{2V^4} \right)_{K'} n_e + \mathcal{O}(V^{-4}). \quad (52)$$

Disregarding the term $()_{K'}$ since we are assuming $V \gg \vartheta_i$, and remembering that in subsection II C 1 we associated boson W' with SM boson W , we can easily associate:

$$\sqrt{2}G_F \approx \frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}. \quad (53)$$

We note that eq. (53) gives limits for the VEV of one of Higgs triplets. Under assumption $\vartheta_1 \sim \vartheta_2 \ll V$, we can write $G_F \approx \frac{1}{2\sqrt{2}\vartheta_2^2} \left(1 - \frac{\vartheta_1^2}{V^2} \right)$, from which can see that the maximum value of ϑ_2^2 is achieved when we consider $\frac{\vartheta_1^2}{V^2} = 0$, in which replacing $G_F = 1.16637(1) \times 10^{-5} \text{GeV}^{-2}$ leads to

$$\vartheta_2 \lesssim 174.105 \text{ GeV} \quad . \quad (54)$$

B. Neutral Current

The lagrangian for neutrino elastic-scattering with fermions $f = e, u, d$ through NC is given by:

$$\begin{aligned}-\mathcal{L}_{int}^{NC} = & \bar{f}(p_1)\gamma^\mu \left(g_{z'L}^f + g_{z'R}^f \right) f(p_2) \frac{-ig_{\mu\lambda}}{(p_2 - p_1)^2 - M_{z'}^2} \bar{\nu}_{\ell L}(p_3)\gamma^\lambda g_{\nu z'}\nu_{\ell L}(p_4) \\ & + \bar{f}(p_1)\gamma^\mu \left(g_{z'L}^f + g_{z'R}^f \right) f(p_2) \frac{-ig_{\mu\lambda}}{(p_2 - p_1)^2 - M_z^2} \bar{\nu}_{\ell L}(p_3)\gamma^\lambda g_{\nu z}\nu_{\ell L}(p_4) \\ & + \bar{f}(p_1)\gamma^\mu \left(g_{k'L}^f + g_{k'R}^f \right) f(p_2) \frac{-ig_{\mu\lambda}}{(p_2 - p_1)^2 - M_{k'}^2} \bar{\nu}_{\ell L}(p_3)\gamma^\lambda g_{\nu k'}\nu_{\ell L}(p_2).\end{aligned}\quad (55)$$

³ Note from eq. (47) that only left-handed leptons interact with neutrinos, as in SM.

For low energies, we have that $M_{k'}^2, M_z^2, M_{z'}^2 \gg (p_2 - p_1)^2$ with $p_3 = p_4 = p$ and eq. (55) can be written as:

$$\begin{aligned} -\mathcal{L}_{eff}^{NC} &\approx \frac{G_{\nu z'}}{M_{z'}^2} \left\langle \bar{f}(p_1) \gamma^\mu \left(g_{z'L}^f + g_{z'R}^f \right) f(p_2) \right\rangle \bar{\nu}_{\ell L} \gamma_\mu \nu_{\ell L} \\ &+ \frac{G_{\nu z}}{M_z^2} \left\langle \bar{f}(p_1) \gamma^\mu \left(g_{zL}^f + g_{zR}^f \right) f(p_2) \right\rangle \bar{\nu}_{\ell L} \gamma_\mu \nu_{\ell L} \\ &+ \frac{G_{\nu k'}}{M_{k'}^2} \left\langle \bar{f}(p_1) \gamma^\mu \left(g_{k'L}^f + g_{k'R}^f \right) f(p_2) \right\rangle \bar{\nu}_{\ell L} \gamma_\mu \nu_{\ell L}. \end{aligned} \quad (56)$$

Following the same procedure of section III A we obtain:

$$\mathcal{L}_{eff}^{NC} \approx - \sum_{P=L,R} \left(g_{z'P}^f \frac{G_{\nu z'}}{M_{z'}^2} + g_{zP}^f \frac{G_{\nu z}}{M_z^2} + g_{k'P}^f \frac{G_{\nu k'}}{M_{k'}^2} \right) \frac{1}{2} n_f \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L}. \quad (57)$$

1. Leptons sector

From eqs. (40) and (33), we obtain that for the known neutral leptons:

$$\begin{aligned} \frac{g_x}{3} \bar{\nu}_{\ell L} \gamma^\mu \nu_{\ell L} B_\mu &= \bar{\nu}_{\ell L} \gamma^\mu \nu_{\ell L} \left[-\frac{g}{3} S_W A_\mu + \left(\frac{g}{3} T_W^2 C_W + \frac{g_x}{3} \beta_1 \right) Z_\mu^0 \right. \\ &\quad \left. - \frac{g_x}{3} \left(\frac{1}{\sqrt{3}} T_W - \beta_2 \right) K_{R\mu}^{'0} \right], \end{aligned} \quad (58)$$

$$\frac{g}{2} \bar{\nu}_{\ell L} \gamma^\mu \nu_{\ell L} W_3^\mu = \bar{\nu}_{\ell L} \gamma^\mu \nu_{\ell L} \left[\frac{g}{2} S_W A_\mu - \frac{g\vartheta_1}{2V} Z_\mu^{'0} + \frac{g(C_W + \beta_3)}{2} Z_\mu^0 + \frac{g\beta_4}{2} K_{R\mu}^{'0} \right], \quad (59)$$

$$\begin{aligned} \frac{-g}{2\sqrt{3}} \bar{\nu}_{\ell L} \gamma^\mu \nu_{\ell L} W_8^\mu &= \bar{\nu}_{\ell L} \gamma^\mu \nu_{\ell L} \left[-\frac{g}{6} S_W A_\mu - \frac{g\vartheta_1}{2V} Z_\mu^{'0} + \left(\frac{g}{6} \frac{S_W^2}{C_W} - \frac{g\beta_5}{2\sqrt{3}} \right) Z_\mu^0 \right. \\ &\quad \left. + \frac{g}{2\sqrt{3}} \left(\frac{1}{t} T_W - \beta_6 \right) K_{R\mu}^{'0} \right]. \end{aligned} \quad (60)$$

By eqs. (58), (59) and (60) we obtain that vertex interactions with neutrinos can be written as:

$$\bar{\nu}_{\ell L} \gamma^\mu \nu_{\ell L} A_\mu \propto 0, \quad (61)$$

$$\bar{\nu}_{\ell L} \gamma^\mu \nu_{\ell L} Z_\mu^{'0} \propto -\frac{g\vartheta_1}{V} \equiv G_{\nu Z'}, \quad (62)$$

$$\bar{\nu}_{\ell L} \gamma^\mu \nu_{\ell L} Z_\mu^0 \propto \frac{1}{2} g C_W^{-1} + \eta_1 \equiv G_{\nu Z}, \quad (63)$$

$$\bar{\nu}_{\ell L} \gamma^\mu \nu_{\ell L} K_{R\mu}^{'0} \propto \left(\frac{3g - 2g_x t}{6\sqrt{3}t} \right) T_W + \eta_2 \equiv G_{\nu K'}, \quad (64)$$

where

$$\begin{aligned} \eta_1 &= \frac{-4gt C_W^2 \vartheta_1^2 + g_x (1 - 2S_W^2) \vartheta_2^2}{8tV^2 C_W^5}, \\ \eta_2 &= \frac{gt (1 - 4C_W^2) \vartheta_1^2}{2\sqrt{3}V^2 C_W S_W} - \frac{(-gt^3 + 2gt^3 C_W^2 + 8gt^3 C_W^4 + 6g_x S_W^4) \vartheta_2^2}{24\sqrt{3}t^2 V^2 C_W^5 S_W}. \end{aligned}$$

We note from eq. (61) that neutrinos does not interact electrically, as expected. For charged leptons, from eqs. (40) and (33) we obtain:

$$\begin{aligned} \frac{g_x}{3} \bar{\ell}_L \gamma^\mu \ell_L B_\mu &= \bar{\ell}_L \gamma^\mu \ell_L \left[-\frac{g}{3} S_W A_\mu + \left(\frac{g}{3} T_W^2 C_W + \frac{g_x}{3} \beta_1 \right) Z_\mu^0 \right. \\ &\quad \left. - \frac{g_x}{3} \left(\frac{1}{\sqrt{3}} T_W - \beta_2 \right) K_{R\mu}^{'0} \right], \end{aligned} \quad (65)$$

$$-\frac{g}{2} \bar{\ell}_L \gamma^\mu \ell_L W_3^\mu = \bar{\ell}_L \gamma^\mu \ell_L \left[-\frac{g}{2} S_W A_\mu + \frac{g\vartheta_1}{2V} Z_\mu^{'0} - \frac{g(C_W + \beta_3)}{2} Z_\mu^0 - \frac{g\beta_4}{2} K_{R\mu}^{'0} \right], \quad (66)$$

$$\begin{aligned} \frac{-g}{2\sqrt{3}} \bar{\ell}_L \gamma^\mu \ell_L W_8^\mu &= \bar{\ell}_L \gamma^\mu \ell_L \left[\frac{-g}{6} S_W A_\mu - \frac{g\vartheta_1}{2V} Z_\mu'^0 + \left(\frac{g}{6} \frac{S_W^2}{C_W} - \frac{g\beta_5}{2\sqrt{3}} \right) Z_\mu^0 \right. \\ &\quad \left. + \frac{g}{2\sqrt{3}} \left(\frac{1}{t} T_W - \beta_6 \right) K_{R\mu}'^0 \right], \end{aligned} \quad (67)$$

$$\begin{aligned} g_x \bar{\ell}_R \gamma^\mu \ell_R B_\mu &= \bar{\ell}_R \gamma^\mu \ell_R \left[-g S_W A_\mu + (g T_W^2 C_W + g_x \beta_1) Z_\mu^0 \right. \\ &\quad \left. - g_x \left(\frac{1}{\sqrt{3}} T_W - \beta_2 \right) K_{R\mu}'^0 \right], \end{aligned} \quad (68)$$

and therefore:

$$\bar{\ell} \gamma^\mu \ell A_\mu \propto -g S_W, \quad (69)$$

$$\bar{\ell}_L \gamma^\mu \ell_L Z_\mu'^0 \propto 0 \equiv g_{z'L}^\ell = g_{z'R}^\ell, \quad (70)$$

$$\bar{\ell}_L \gamma^\mu \ell_L Z_\mu^0 \propto \frac{1}{2} g (-1 + T_W^2) C_W + \eta_3 \equiv g_{zL}^\ell, \quad (71)$$

$$\bar{\ell}_R \gamma^\mu \ell_R Z_\mu^0 \propto g T_W^2 C_W + \eta_5 \equiv g_{zR}^\ell, \quad (72)$$

$$\bar{\ell}_L \gamma^\mu \ell_L K_{R\mu}'^0 \propto \frac{1}{6\sqrt{3}t} (3g - 2tg_x) T_W + \eta_4 \equiv g_{k'L}^\ell, \quad (73)$$

$$\bar{\ell}_R \gamma^\mu \ell_R K_{R\mu}'^0 \propto -\frac{g_x}{\sqrt{3}} T_W + \eta_6 \equiv g_{k'R}^\ell, \quad (74)$$

where

$$\begin{aligned} \eta_3 &= \frac{(-1 + 2C_W^2) g_x \vartheta_2^2}{8tV^2 C_W^5}, \\ \eta_4 &= \frac{(gt^3 (1 + 2C_W^2)^2 - 12gt^3 S_W^2 C_W^2 - 6g_x S_W^4)}{24\sqrt{3}t^2 V^2 C_W^5 S_W}, \\ \eta_5 &= -\frac{g_x \vartheta_2^2}{4tV^2 C_W^3} T_W^2, \\ \eta_6 &= -\frac{\sqrt{3}g_x \vartheta_2^2}{4t^2 V^2 C_W^2} T_W^3, \end{aligned}$$

and, again, $t = g_x/g$. We note that by eq. (69) we can make the association $gS_W = |e|$. Then for $f = e$, eqs. (62)-(64) and (70)-(74) lead to:

$$\begin{aligned} \mathcal{L}_{eff-e}^{NC} &\approx - \sum_{P=L,R} \frac{1}{2} \left(g_{z'P}^e \frac{G_{\nu z'}}{M_{z'}^2} + g_{zP}^e \frac{G_{\nu z}}{M_z^2} + g_{k'P}^e \frac{G_{\nu k'}}{M_{k'}^2} \right) n_e \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L} \\ &\approx - \left\{ \left[\frac{T_W^4}{144t^2 g_x^2 V^2} (3g - 2tg_x)^2 + \frac{T_W^2}{8V^2} (1 - T_W^2) \right. \right. \\ &\quad \left. \left. + \frac{1}{2} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) (1 - 2C_W^2) \right]_L \right. \\ &\quad \left. + \left[\frac{T_W^4 (2tg_x - 3g)}{24tg_x V^2} - \frac{T_W^4}{4V^2} + \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) S_W^2 \right]_R \right\} n_e \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L}. \end{aligned} \quad (75)$$

Since intermediate neutral bosons in eq. (55) does not distinguish between different lepton flavors, the interaction through NC with electron is described by the following effective potential.

$$V_{NC}^e = V_{NC}^\mu = V_{NC}^\tau = V_{NC}^\ell, \quad (76)$$

$$= V_{NC}^{\ell L} + V_{NC}^{\ell R}, \quad (77)$$

where

$$V_{NC}^{\ell L} = \left[\frac{T_W^4}{144t^2 g_x^2 V^2} (3g - 2tg_x)^2 + \frac{T_W^2}{8V^2} (1 - T_W^2) \right]$$

$$+ \frac{1}{2} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{V^2\vartheta_2^2} \right) (1 - 2C_W^2) \Big] n_e, \quad (78)$$

$$V_{NC}^{\ell R} = \left[\frac{T_W^4 (2tg_x - 3g)}{24tg_x V^2} - \frac{T_W^4}{4V^2} + \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) S_W^2 \right] n_e, \quad (79)$$

and index ℓ refers to neutrino flavor. We note that the potential through CC comes from interactions of electron neutrinos with left-handed electrons, while the effective potential through NC comes from left and right-handed electrons.

Considering both NC and CC, we can write the effective potential felt by neutrinos as $V^\ell = V^{\ell L} + V^{\ell R}$ where

$$V^{\ell L} = \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) \delta_{e\ell} n_e + V_{NC}^{\ell L}, \quad (80)$$

$$V^{\ell R} = V_{NC}^{\ell R}. \quad (81)$$

Comparing with SM expression for such potential:

$$\mathbf{V}_{NC}^\ell = -\sqrt{2}G_F \left(\frac{1}{2} - 2S_W^2 \right) n_e, \quad \mathbf{V}_{CC}^e = \sqrt{2}G_F n_e, \quad (82)$$

we can find that

$$V^{\ell L} = \mathbf{V}^{\ell L} + \left[\frac{T_W^4}{144t^2g_x^2V^2} (3g - 2tg_x)^2 + \frac{T_W^2}{8V^2} (1 - T_W^2) \right] n_e, \quad (83)$$

$$V^{\ell R} = \mathbf{V}_{NC}^{\ell R} + \left[\frac{T_W^4 (2tg_x - 3g)}{24tg_x V^2} - \frac{T_W^4}{4V^2} \right] n_e, \quad (84)$$

then, the new terms beyond SM [] can be associated with the parameters ε 's in NSI [45]. So, in the approximation $\left(\frac{\vartheta_i}{V}\right)^n \approx 0$, for $n > 2$ we obtain:

$$\varepsilon_{\ell\ell}^{eL} \approx \frac{(1 - 2S_W^2) \vartheta_2^2}{8V^2 C_W^4}, \quad (85)$$

$$\varepsilon_{\ell\ell}^{eR} \approx -\frac{S_W^2 (1 + 2S_W^2) \vartheta_2^2}{4V^2 C_W^4}. \quad (86)$$

We note that on limit $V \rightarrow \infty$ we recover SM. The NSI are a sub-leading interaction, as expected. By eq. (85) and eq. (86) we obtain: $\varepsilon_{\ell\ell}^{eR} \approx -2S_W^2 \varepsilon_{\ell\ell}^{eL} - \frac{\vartheta_2^2}{V^2} T_W^4$.

2. Quarks sector

For the quarks of first family, Lagrangian density in eq. (43) describes the interactions with gauge bosons $W_{3\mu}$, $W_{8\mu}$ and B_μ , then by eqs. (33) and (34) we obtain the following interactions for quarks up:

$$-\frac{g_x}{3} \overline{u_L} \gamma^\mu u_L B_\mu = \overline{u_L} \gamma^\mu u_L \left[\frac{g}{3} S_W A_\mu - \frac{g_x}{3} \left(\frac{1}{t} T_W^2 C_W + \beta_1 \right) Z_\mu^0 + \frac{g_x}{3} \left(\frac{1}{\sqrt{3}} T_W - \beta_2 \right) K_{R\mu}^{'0} \right], \quad (87)$$

$$\frac{g}{2} \overline{u_L} \gamma^\mu u_L W_3^\mu = \overline{u_L} \gamma^\mu u_L \left[\frac{g}{2} S_W A_\mu - \frac{g\vartheta_1}{2V} Z_\mu^{'0} + \frac{g(C_W + \beta_3)}{2} Z_\mu^0 + \frac{g\beta_4}{2} K_{R\mu}^{'0} \right], \quad (88)$$

$$\frac{-g}{2\sqrt{3}} \overline{u_L} \gamma^\mu u_L W_8^\mu = \overline{u_L} \gamma^\mu u_L \left[\frac{-g}{6} S_W A_\mu - \frac{g\vartheta_1}{2V} Z_\mu^{'0} + \frac{g}{2\sqrt{3}} \left(\frac{1}{\sqrt{3}} T_W S_W - \beta_5 \right) Z_\mu^0 + \frac{g}{2\sqrt{3}} \left(\frac{1}{t} T_W - \beta_6 \right) K_{R\mu}^{'0} \right], \quad (89)$$

$$-\frac{2g_x}{3} \overline{u_R} \gamma^\mu u_R B_\mu = \overline{u_R} \gamma^\mu u_R \left[\frac{2g}{3} S_W A_\mu - \frac{2g_x}{3} \left(\frac{1}{t} T_W^2 C_W + \beta_1 \right) Z_\mu^0 + \frac{2g_x}{3} \left(\frac{1}{t} T_W - \beta_6 \right) K_{R\mu}^{'0} \right]. \quad (90)$$

The couplings quark-quark-boson for the first family are given by:

$$\overline{u_L}\gamma^\mu u_L A_\mu \propto \frac{2}{3}gS_W, \quad (91)$$

$$\overline{u_R}\gamma^\mu u_R A_\mu \propto \frac{2}{3}gS_W, \quad (92)$$

$$\overline{u_L}\gamma^\mu u_L Z'_\mu{}^0 \propto -\frac{g\vartheta_1}{V} \equiv g_{z'L}^u, \quad (93)$$

$$\overline{u_R}\gamma^\mu u_R Z'_\mu{}^0 \propto 0 \equiv g_{z'R}^u, \quad (94)$$

$$\overline{u_L}\gamma^\mu u_L Z_\mu{}^0 \propto \frac{1}{6}g(3 - T_W^2)C_W + \zeta_1 \equiv g_{zL}^u, \quad (95)$$

$$\overline{u_R}\gamma^\mu u_R Z_\mu{}^0 \propto -\frac{2}{3}gT_W^2C_W + \zeta_3 \equiv g_{zR}^u, \quad (96)$$

$$\overline{u_L}\gamma^\mu u_L K_{R\mu}^{'0} \propto \frac{1}{6\sqrt{3}t}(3g + 2tg_x)T_W \equiv g_{k'L}^u, \quad (97)$$

$$\overline{u_R}\gamma^\mu u_R K_{R\mu}^{'0} \propto \frac{2}{3\sqrt{3}}g_xT_W + \zeta_4 \equiv g_{k'R}^u, \quad (98)$$

where

$$\begin{aligned} \zeta_1 &= \frac{g_x(-12C_W^4\vartheta_1^2 + (1 + 2C_W^2)\vartheta_2^2)}{24tV^2C_W^5}, \\ \zeta_2 &= \frac{12gt^3C_W^4(1 - 4C_W^2)\vartheta_1^2 + (gt^3(1 - 2C_W^2 - 8C_W^4) + 6g_xS_W^4)\vartheta_2^2}{24\sqrt{3}t^2V^2C_W^5S_W}, \\ \zeta_3 &= \frac{g}{6}\frac{S_W^2\vartheta_2^2}{C_W^5V^2}, \\ \zeta_4 &= \frac{g_xS_W^3\vartheta_2^2}{2\sqrt{3}t^2V^2C_W^5}. \end{aligned} \quad (99)$$

We note that eqs. (91) and (92) reflect the fact that quarks interact electrically through photons with coupling constant $Q_f \sin \theta_W$, as in SM. The effective lagrangian at low energies for neutrino interaction with quarks up through neutral currents are given by eq. 57 with $f = u$:

$$\begin{aligned} \mathcal{L}_{quark,u}^{NC} &\approx -\frac{1}{2} \sum_{P=L,R} \left(g_{z'P}^u \frac{G_{\nu z'}}{M_{z'}^2} + g_{zP}^u \frac{G_{\nu z}}{M_z^2} + g_{k'P}^u \frac{G_{\nu k'}}{M_{k'}^2} \right) n_u \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L} \\ &\approx - \left\{ \left[\frac{1}{24V^2} (3 + T_W^4) + \frac{T_W^4}{144t^4V^2} (9 - 4t^4) \right. \right. \\ &\quad + \left. \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) \left(\frac{1}{2} - \frac{2}{3}S_W^2 \right) - \frac{\vartheta_1^2}{4V^2\vartheta_2^2} \right]_L \\ &\quad + \left. \left[\frac{T_W^4}{6V^2} + \frac{T_W^4(3g - 2tg_x)}{36tg_xV^2} - \frac{2}{3} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) S_W^2 \right]_R \right\} n_u \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L}, \end{aligned} \quad (100)$$

where n_u is the up-quarks average density.

SM predictions, using result of eq.(53), can be written as:

$$\mathbf{V}_{NC}^u = \mathbf{V}_{NC}^{uL} + \mathbf{V}_{NC}^{uR} = \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) \left(\frac{1}{2} - \frac{4}{3}S_W^2 \right) n_u, \quad (101)$$

where

$$\mathbf{V}_{NC}^{uL} = \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) \left(\frac{1}{2} - \frac{2}{3}S_W^2 \right) n_u, \quad (102)$$

$$\mathbf{V}_{NC}^{uR} = -\frac{2}{3} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) S_W^2 n_u. \quad (103)$$

By comparison, we obtain:

$$V_{NC}^{uL} \approx V_{NC}^{uL} + \left[\frac{1}{24V^2} (3 + T_W^4) + \frac{T_W^4}{144t^4V^2} (9 - 4t^4) - \frac{\vartheta_1^2}{4V^2\vartheta_2^2} \right] n_u, \quad (104)$$

$$V_{NC}^{uR} \approx V_{NC}^{uR} + \left[\frac{T_W^4}{6V^2} + \frac{T_W^4 (3g - 2tg_x)}{36tg_xV^2} \right] n_u. \quad (105)$$

Then we can say that $\varepsilon_{\ell\ell}^u = \varepsilon_{\ell\ell}^{uL} + \varepsilon_{\ell\ell}^{uR}$ where

$$\varepsilon_{\ell\ell}^{uL} \approx -\frac{\vartheta_1^2}{2V^2} + \frac{\vartheta_2^2}{24V^2C_W^4} (9 - 8S_W^2), \quad (106)$$

$$\varepsilon_{\ell\ell}^{uR} \approx \frac{\vartheta_2^2}{6V^2} \frac{S_W^2}{C_W^4}, \quad (107)$$

again, we obtain universal NSI, as for the electrons. We note that $\varepsilon_{\ell\ell}^{uL} = -\frac{\vartheta_1^2}{2V^2} + \frac{3\vartheta_2^2}{8V^2C_W^4} - 2\varepsilon_{\ell\ell}^{uR}$ and in the limit $V \rightarrow \infty$ we recover SM.

For quarks down by eq. (43) and (33) we obtain that:

$$\begin{aligned} -\frac{g_x}{3} \overline{d_L} \gamma^\mu d_L B_\mu &= \overline{d_L} \gamma^\mu d_L \left[\frac{g}{3} S_W A_\mu - \frac{g_x}{3} \left(\frac{1}{t} T_W^2 C_W + \beta_1 \right) Z_\mu^0 \right. \\ &\quad \left. + \frac{g_x}{3} \left(\frac{1}{\sqrt{3}} T_W - \beta_2 \right) K_{R\mu}^{\prime 0} \right], \end{aligned} \quad (108)$$

$$-\frac{g}{2} \overline{d_L} \gamma^\mu d_L W_3^\mu = \overline{d_L} \gamma^\mu d_L \left[-\frac{gS_W}{2} A_\mu + \frac{g\vartheta_1}{2V} Z_\mu^{\prime 0} - \frac{g(C_W + \beta_3)}{2} Z_\mu^0 - \frac{g\beta_4}{2} K_{R\mu}^{\prime 0} \right], \quad (109)$$

$$\begin{aligned} \frac{-g}{2\sqrt{3}} \overline{d_L} \gamma^\mu d_L W_8^\mu &= \overline{d_L} \gamma^\mu d_L \left[-\frac{gS_W}{6} A_\mu + \frac{g}{2\sqrt{3}} \left(\frac{1}{\sqrt{3}} T_W S_W - \beta_5 \right) Z_\mu^0 \right. \\ &\quad \left. + -\frac{g\vartheta_1}{2V} Z_\mu^{\prime 0} + \frac{g}{2\sqrt{3}} \left(\frac{1}{t} T_W - \beta_6 \right) K_{R\mu}^{\prime 0} \right], \end{aligned} \quad (110)$$

$$\begin{aligned} \frac{g_x}{3} \overline{d_R} \gamma^\mu d_R B_\mu &= \overline{d_R} \gamma^\mu d_R \left[-\frac{gS_W}{3} A_\mu + \frac{g_x}{3} \left(\frac{1}{t} T_W^2 C_W + \beta_1 \right) Z_\mu^0 \right. \\ &\quad \left. + \frac{g_x}{3} \left(-\frac{1}{\sqrt{3}} T_W + \beta_2 \right) K_{R\mu}^{\prime 0} \right]. \end{aligned} \quad (111)$$

Then the couplings quark-quark-boson for the first family are given by:

$$\overline{d_L} \gamma^\mu d_L A_\mu \propto -\frac{1}{3} g S_W, \quad (112)$$

$$\overline{d_R} \gamma^\mu d_R A_\mu \propto -\frac{1}{3} g S_W, \quad (113)$$

$$\overline{d_L} \gamma^\mu d_L Z_\mu^{\prime 0} \propto 0 \equiv g_{z'L}^d, \quad (114)$$

$$\overline{d_R} \gamma^\mu d_R Z_\mu^{\prime 0} \propto 0 \equiv g_{z'R}^d, \quad (115)$$

$$\overline{d_L} \gamma^\mu d_L Z_\mu^0 \propto -\frac{1}{6} g (3 + T_W^2) C_W + \zeta_5 \equiv g_{zL}^d, \quad (116)$$

$$\overline{d_R} \gamma^\mu d_R Z_\mu^0 \propto \frac{g}{3} T_W^2 C_W + \zeta_7 \equiv g_{zR}^d, \quad (117)$$

$$\overline{d_L} \gamma^\mu d_L K_{R\mu}^{\prime 0} \propto \frac{1}{6\sqrt{3}t} (3g + 2tg_x) T_W + \zeta_6 \equiv g_{k'L}^d, \quad (118)$$

$$\overline{u_R} \gamma^\mu u_R K_{R\mu}^{\prime 0} \propto -\frac{1}{3\sqrt{3}} g_x T_W + \zeta_8 \equiv g_{k'R}^d, \quad (119)$$

where

$$\zeta_5 = \frac{g\vartheta_2^2}{24V^2C_W^5} (3 - 2S_W^2),$$

$$\begin{aligned}
\zeta_6 &= \frac{(-1 + 3C_W^2 + 6C_W^4 - 8C_W^6)}{24\sqrt{3}V^2C_W^5S_W^3}, \\
\zeta_7 &= -\frac{gS_W^2\vartheta_2^2}{12V^2C_W^5}, \\
\zeta_8 &= -\frac{g_xS_W^3\vartheta_2^2}{4\sqrt{3}t^2V^2C_W^5},
\end{aligned} \tag{120}$$

then by eq. (57) for $f = d$ we obtain the following effective lagrangian for NC:

$$\begin{aligned}
\mathcal{L}_{quark,d}^{NC} &\approx - \left(g_{z'V}^d \frac{G_{\nu z'}}{M_{z'}^2} + g_{zV}^d \frac{G_{\nu z}}{M_z^2} + g_{k'V}^d \frac{G_{\nu k'}}{M_{k'}^2} \right) n_d \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L} \\
&\approx - \left\{ \left[\frac{(3S_W^2 - 2S_W^4)}{24V^2C_W^4} + \frac{(9 - 4t^4)}{144t^4V^2} T_W^4 \right. \right. \\
&\quad \left. \left. + \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) \left(-\frac{1}{2} + \frac{1}{3}S_W^2 \right) \right]_L \right. \\
&\quad \left. + \left[-\frac{S_W^2}{24V^2C_W^4} + \frac{1}{3} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) S_W^2 \right]_R \right\} n_d \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L},
\end{aligned} \tag{121}$$

$$+ \left[-\frac{S_W^2}{24V^2C_W^4} + \frac{1}{3} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) S_W^2 \right]_R \Big\} n_d \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L}, \tag{122}$$

and the effective potential felt by neutrinos when crossing a medium composed by a density n_d of *down* quarks is $V_{NC}^d = V_{NC}^{dL} + V_{NC}^{dR}$ where

$$V_{NC}^{dL} \approx \left[\frac{(3S_W^2 - 2S_W^4)}{24V^2C_W^4} + \frac{(9 - 4t^4)}{144t^4V^2} T_W^4 + \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) \left(-\frac{1}{2} + \frac{1}{3}S_W^2 \right) \right] n_d, \tag{123}$$

$$V_{NC}^{dR} \approx \left[-\frac{S_W^2}{24V^2C_W^4} + \frac{1}{3} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) S_W^2 \right] n_d. \tag{124}$$

Then we can easily see that in SM the NC effective potential for neutrinos in a d-quark medium, using result of eq. 53, will be given by:

$$\begin{aligned}
V_{NC}^d &= V_{NC}^{dL} + V_{NC}^{dR} \approx - \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) \left(\frac{1}{2} - \frac{2}{3}S_W^2 \right) n_d, \\
V_{NC}^{dL} &= \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) \left(-\frac{1}{2} + \frac{1}{3}S_W^2 \right) n_d,
\end{aligned} \tag{125}$$

$$V_{NC}^{dR} = \frac{1}{3} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) S_W^2 n_d. \tag{126}$$

Then from eq. (123)- (126), we obtain:

$$V_{CN}^{dL} \approx V_{NC}^{dL} + \left[\frac{(3S_W^2 - 2S_W^4)}{24V^2C_W^4} + \frac{(9 - 4t^4)}{144t^4V^2} T_W^4 \right] n_d, \tag{127}$$

$$V_{NC}^{dR} \approx V_{NC}^{dR} - \frac{S_W^2}{24V^2C_W^4} n_d, \tag{128}$$

and neglecting terms of order $\left(\frac{\vartheta_i}{V}\right)^n$, for $n > 2$ we obtain that $\varepsilon_{\ell\ell}^d = \varepsilon_{\ell\ell}^{dL} + \varepsilon_{\ell\ell}^{dR}$ where

$$\varepsilon_{\ell\ell}^{dL} \approx \frac{\vartheta_2^2}{24V^2C_W^4} (3 - 2S_W^2), \tag{129}$$

$$\varepsilon_{\ell\ell}^{dR} \approx -\frac{S_W^2\vartheta_2^2}{12V^2C_W^4}. \tag{130}$$

Then we obtain $\varepsilon_{\ell\ell}^{dL} \approx \frac{\vartheta_2^2}{8V^2C_W^4} + \varepsilon_{\ell\ell}^{dR}$. Note that again in limit $V \rightarrow \infty$ we recover the SM.

TABLE I: Values for NSI in 331 model and experimental limits

Modelo331		Exp. 90% C.L. [46]
$\varepsilon_{\ell\ell}^{eL} \approx \frac{(1-2S_W^2)\vartheta_2^2}{8V^2C_W^4}$	$0.114 \left(\frac{\vartheta_2^2}{V^2} \right)$	$-0.07 < \varepsilon_{ee}^{eL} < 0.11$ $-0.025 < \varepsilon_{\mu\mu}^{eL} < 0.03$ $-0.6 < \varepsilon_{\tau\tau}^{eL} < 0.4$
$\varepsilon_{\ell\ell}^{eR} \approx -2S_W^2\varepsilon_{\ell\ell}^{eL} - \frac{\vartheta_2^2}{V^2}T_W^4$	$0.143 \left(\frac{\vartheta_2^2}{V^2} \right)$	$-1 < \varepsilon_{ee}^{eR} < 0.5$ $-0.027 < \varepsilon_{\mu\mu}^{eR} < 0.03$ $-0.4 < \varepsilon_{\tau\tau}^{eR} < 0.6$
$\varepsilon_{\ell\ell}^{uL} \approx -\frac{\vartheta_1^2}{2V^2} + \frac{\vartheta_2^2}{24V^2C_W^4} (9 - 8S_W^2)$	$0.50 \left(\frac{\vartheta_2^2 - \vartheta_1^2}{V^2} \right)$	$-1 < \varepsilon_{ee}^{uL} < 0.3$ $ \varepsilon_{\mu\mu}^{uL} < 0.003$ $ \varepsilon_{\tau\tau}^{uL} < 1.4$
$\varepsilon_{\ell\ell}^{uR} \approx \frac{\vartheta_2^2}{6V^2} \frac{S_W^2}{C_W^4}$	$0.065 \left(\frac{\vartheta_2^2}{V^2} \right)$	$-0.4 < \varepsilon_{ee}^{uR} < 0.7$ $-0.008 < \varepsilon_{\mu\mu}^{uR} < 0.003$ $ \varepsilon_{\tau\tau}^{uR} < 3$
$\varepsilon_{\ell\ell}^{dL} \approx \frac{\vartheta_2^2}{24V^2C_W^4} (3 - 2S_W^2)$	$0.179 \left(\frac{\vartheta_2^2}{V^2} \right)$	$-0.3 < \varepsilon_{ee}^{dL} < 0.3$ $ \varepsilon_{\mu\mu}^{dL} < 0.003$ $ \varepsilon_{\tau\tau}^{dL} < 1.1$
$\varepsilon_{\ell\ell}^{dR} \approx -\frac{S_W^2\vartheta_2^2}{12V^2C_W^4}$	$-0.033 \left(\frac{\vartheta_2^2}{V^2} \right)$	$-0.6 < \varepsilon_{ee}^{dR} < 0.5$ $-0.008 < \varepsilon_{\mu\mu}^{dR} < 0.015$ $ \varepsilon_{\tau\tau}^{dR} < 6$

IV. RESULTS

In last sections we saw that in 331 model we chosed, all NSI parameters are universal and diagonal, and will not affect oscillation experiments. However, measurements of cross-section will be sensitive to such parameters, through modifications on g_i^α [28]. We will now compare our results with those obtained in cross-section measurements. We will assume $\sin^2 \theta_W = 0.23149(13)$.

In table I we can see that constrains in $\varepsilon_{\ell\ell}^{eP}$ lead to $V^2 > 5.3\vartheta_2^2$, while the constrains in $\varepsilon_{\ell\ell}^{uP}$ lead to $V^2 > 21.7\vartheta_2^2$, and the constrains in $\varepsilon_{\ell\ell}^{dP}$ ($|\varepsilon_{\mu\mu}^{dL}| < 0.003$) lead to $V^2 > 60\vartheta_2^2$. If ϑ_2 has its maximum value of 174.105 GeV then $V \gtrsim 1.3$ TeV. We note also that by $|\varepsilon_{\mu\mu}^{uL}| < 0.003$ we obtain $|\vartheta_2^2 - \vartheta_1^2| < 0.006V^2$, then for $V \sim 1.3$ TeV and $\vartheta_2 = 174$ GeV we obtain $142\text{GeV} < \vartheta_1 < 201\text{GeV}$. We therefore can not predict any hierarchy to the VEV's ϑ_1 and ϑ_2 . Based on that results, we obtain the following inferior limits for the new Gauge bosons masses:

$$\begin{aligned}
M_{K_I} &= M_{Z'} > 610 \text{ GeV}, \\
M_{K'} &> 613 \text{ GeV}, \\
M_{K_R} &> 740 \text{ GeV}.
\end{aligned}$$

V. CONCLUSION

We presented in this work a procedure to show that models with extended Gauge symmetries $SU(3)_C \times SU(3)_L \times U(1)_X$ can lead to neutrino non-standard interactions, respecting the Standard Model Gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$, without spoiling the available experimental data and reproducing the known phenomenology at low energies. We also have shown that with an assumption about a mass hierarchy for the Higgs triplets VEV's we could qualitatively address the mass hierarchy problem in standard model. Finally we obtained limits for the triplets VEV's based on limits for NSI in cross-section experiments.

We believe that the class of model presented here is an interesting theoretical possibility to look for new physics beyond SM. We restrained our work to a simple scenario, but flavor-changing interactions can be naturally introduced in the model, leading to new constraints on NSI.

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